

**William I. Fine Theoretical Physics Institute
University of Minnesota**

FTPI-MINN-08/40

UMN-TH-2722/08

October 2008

Spontaneous decay of a metastable domain wall

A. Monin

School of Physics and Astronomy, University of Minnesota,
Minneapolis, MN 55455, USA,
and

M.B. Voloshin

William I. Fine Theoretical Physics Institute, University of Minnesota,
Minneapolis, MN 55455, USA
and

Institute of Theoretical and Experimental Physics, Moscow, 117218, Russia

Abstract

We consider the decay of a metastable domain wall. The transition proceeds through quantum tunneling, and we calculate in arbitrary number of dimensions the preexponential factor multiplying the leading semiclassical exponential expression for the rate of the process. We find that the effect of the motion in transverse directions reduces to a renormalization of the tension of the edge of the wall in the semiclassical exponent. This behavior is similar to the one previously found for breaking of a metastable string. However this simple property is generally lost for spontaneous decay of higher-dimensional branes.

1 Introduction

Metastable domain wall solutions arise in models with spontaneously broken approximate symmetry. The existence of such solutions was shown from different points of view, for example, in [1, 2, 3]. The origin of a metastable wall can be illustrated in a model with potential shown in Fig.1, it corresponds to the interpolation between the same vacuum state at two spatial infinities, e.g. at $z = -\infty$ and $z = +\infty$ with the field winding around the ‘peg’ in the potential.

Such configuration is classically stable in the sense that the solution with certain topological number (winding number) can not evolve classically to a solution with different topological number. However, such a transition can proceed due to temperature fluctuation, when the path, corresponding to the solution can be lifted over the barrier or due to quantum tunneling (the motion in a classically forbidden region), hence the domain wall can undergo a decay at certain conditions. In this paper we restrict ourselves to the later case.

The decay of a domain wall is analogous to decay of metastable vacuum[4, 5, 6] in $2 + 1$ dimensions. Indeed, if a hole with area A is created in a wall, the gain in the energy is εA . The barrier, on the other side, that inhibits the process is created by the energy $\mu\ell$, with ℓ being the perimeter of the hole and μ being a tension associated with the interface. Thus, the ‘area’ energy gain exceeds the barrier energy only starting from a critical size of the hole created, i.e. starting with a round hole of radius $R = 2\mu/\varepsilon$. Once a critical piece has nucleated due to tunneling, it expands, converting the domain wall. Therefore the probability of the transition is given by the rate of nucleation of the critical holes in the domain wall. The important difference, addressed in this paper, from the $2 + 1$ dimensional false vacuum decay is that the domain wall can move in the transverse direction(s) and this motion affects the nucleation rate at the level of the preexponential factor.

Recently we have considered a similar problem of calculating the rate of a transition between two states of a string with different tension [7]. It was shown that in a case of complete breaking (transition into nothing) of a string with tension ε the essential effect of the motion in transverse dimensions reduces to a renormalization of mass parameter μ associated with the interface between two states, such that the dependence of the rate on string’s tension has the same form as for two dimensions, where it coincides with well-known one for false vacuum decay or Schwinger process of charged pair production in external

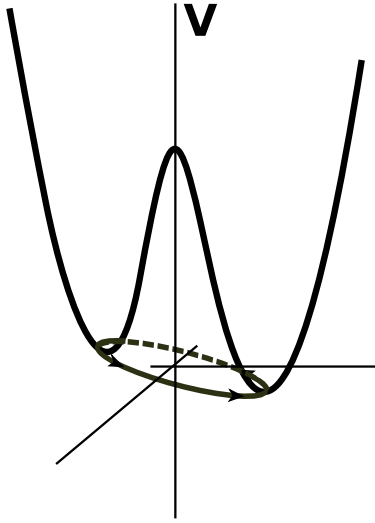


Figure 1: Potential

electromagnetic field [8], expressed in terms of renormalized parameter μ_R

$$\frac{d\Gamma}{d\ell} = \frac{\varepsilon}{2\pi} \exp\left(-\frac{\pi\mu_R^2}{\varepsilon}\right). \quad (1)$$

As a result of the calculation to be presented in this paper we find that a similar behavior also applies to the decay of a metastable wall. Namely the essential effect of the motion of the wall in the transverse dimension(s) reduces to a renormalization of the boundary tension parameter μ in the expression for the false vacuum decay in $2 + 1$ dimensions. It can be noted that such a reduction is not trivial and holds only for the transitions of strings and two-dimensional walls. We have explicitly verified that such behavior is lost in similar transition of higher dimensional branes, where the transverse motion produces effects that are not reduced to a renormalization of the tension of the interface.

The decay rate of a QCD domain wall with tension ε with further creation of the interface with tension μ was considered in Ref. [9] by adapting the expression for metastable vacuum decay in $2 + 1$ dimensions:

$$\frac{d\Gamma}{dA} = \mathcal{P} \exp\left(-\frac{16\pi\mu^3}{3\varepsilon^2}\right), \quad (2)$$

where \mathcal{P} is the preexponential factor. This factor is found from a calculation of the path integral over small deviations from the semiclassical tunneling trajectory. In the limit of small ε the result of such calculation in a $(2+1)$ dimensional theory, can be readily copied

from the corresponding expressions in the equivalent (2+1) dimensional problem of false vacuum decay[6]

$$\mathcal{P}_{d=2+1} = \frac{\mathcal{C}}{\varepsilon^{7/3}}, \quad (3)$$

with \mathcal{C} being a constant independent of wall tension ε , which can not be found from effective description of the theory. In other words, the constant \mathcal{C} is determined by details of the underlying ‘microscopic’ model, but the dependence of the factor (3) on ε is universal.

In this paper we calculate the domain wall decay rate per unit area in arbitrary number of dimensions d , and find the result in a similar form,

$$\frac{d\Gamma}{dA} = \frac{\tilde{\mathcal{C}}}{\varepsilon^{7/3}} \exp\left(-\frac{16\pi\mu_R^3}{3\varepsilon^2}\right), \quad (4)$$

where again the constant $\tilde{\mathcal{C}}$ does not depend on ε , and μ_R is the tension of the interface that includes the renormalization effect of the wall fluctuations in the transverse dimensions.

The material in this paper is organized as follows. In Sec. 2 we formulate the problem in terms of the effective Euclidean-space action, and in Sec. 3 we consider the separation of variables in the relevant path integrals with this action and also find formal expressions for these integrals in terms of products. A Pauli-Villars regularization for the products is introduced in Sec. 4, and the actual calculation is done in Sec. 5. In Sec. 6 we demonstrate that the dependence of the result on the regulator parameter is completely absorbed into the renormalization of the interface tension μ . Finally in Sec. 7 we present our results and conclusions.

2 Euclidean action

The low energy effective action for the problem at hand is given by Nambu-Goto action for two and tree dimensional objects

$$S = \mu \mathcal{A} + \varepsilon V, \quad (5)$$

with V being the world volume of the wall, while \mathcal{A} is the world area of interface. Nontrivial classical solution (bounce), which defines the exponential behavior of the rate is empty sphere with radius

$$R = \frac{2\mu}{\varepsilon}, \quad (6)$$

surrounded by the metastable phase (Fig.2). The action (5) is the low energy effective

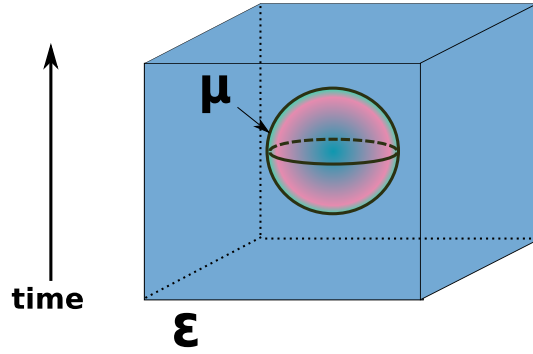


Figure 2: Bounce configuration

expression, and it does not take into account the thickness of the wall, or of the interface. Hence it is valid only while one can neglect the structure of the objects and consider them as having zero thickness. If the thickness of the wall is of order r_0 then the natural mass scale associated with it is $M_0 = 1/r_0$. Therefore one can write the conditions of applicability of the action (5)

$$k \ll M_0, \quad \ell \gg r_0, \quad (7)$$

with k and ℓ being any momentum and length scales in the problem. For instance the radius of the bubble should be much larger than the thickness of the wall

$$M_0 R = \frac{2\mu M_0}{\varepsilon} \gg 1. \quad (8)$$

The probability rate of the transition is given by the imaginary part of the ratio of the partition functions calculated around the bounce and the trivial configurations

$$\frac{d\Gamma}{dA} = \frac{1}{AT} \text{Im} \frac{\mathcal{Z}_{12}}{\mathcal{Z}_1}. \quad (9)$$

The imaginary part of \mathcal{Z}_{12} arises from one negative mode at the bounce configuration. Furthermore, due to three translational zero modes, the numerator in Eq.(9) is proportional to the total world volume AT occupied by the wall, so that the finite quantity is the transition probability per unit time (the rate) and per unit area of the wall.

In order to evaluate the relevant path integrals with the pre-exponential accuracy we use the generalized cylindrical coordinates, with r , θ and φ being the spherical variables in the (t, x, y) space (of the bounce), and z being the transverse coordinate. We consider only one

transverse coordinate, since the effect of each of the extra dimensions factorizes, so that the corresponding generalization is straightforward. We further assume, for definiteness, that the space-time boundary in the (t, x, y) space is a sphere of large radius L , where the boundary condition for the wall is $z(r = L) = 0$. The small deviations of the wall configuration from the bounce can be parametrized by the variation of the bulk: $z(r, \theta, \varphi)$, the radial (f) and the transverse (ζ) shifts of the boundary:

$$r(\theta, \varphi) = R + f(\theta, \varphi) \quad z(R, \theta, \varphi) = \zeta(\theta, \varphi) , \quad (10)$$

In terms of these variables the action (5) can be written in the quadratic approximation in the deviations from the bounce as

$$S_{12} = \frac{4\pi}{3}\varepsilon L^3 + \frac{16\pi\mu^3}{3\varepsilon^2} + \frac{\mu}{2} \int d\Omega \left(g^{ab} \partial_a f \partial_b f - 2f^2 + g^{ab} \partial_a \zeta \partial_b \zeta \right) + \frac{\varepsilon}{2} \int_R^L r^2 dr d\Omega g^{ij} \partial_i z \partial_j z , \quad (11)$$

where the tensor g_{ij} is the Euclidean metric tensor in spherical coordinates ($i, j = r, \theta, \varphi$), while g_{ab} is induced metric on the sphere ($a, b = \theta, \varphi$), $d\Omega = \sin \theta d\theta d\varphi$.

The action around a trivial configuration in the quadratic approximation takes the form

$$S_1 = \frac{4\pi}{3}\varepsilon L^3 + \frac{\varepsilon}{2} \int_0^L r^2 dr d\Omega g^{ij} \partial_i z \partial_j z , \quad (12)$$

3 Separating variables in path integral

One can immediately see that the variable corresponding to the longitudinal variations f of the boundary is decoupled from other variables. This implies that the path integral over f can be considered independently of the integration over other variables and that it enters as a factor in \mathcal{Z}_{12} . On the other hand it is this integral that provides the imaginary part to the partition function, and it is also proportional to the total space-time area AT . Moreover, this path integral is identical to the one entering the problem of false vacuum decay in (2+1) dimensions and we can directly apply the result of that calculation [6]:

$$\frac{1}{AT} \text{Im} \int \mathcal{D}f \exp \left[\frac{\mu}{2} \int d\Omega \left(g^{ab} \partial_a f \partial_b f - 2f^2 \right) \right] = \frac{\mathcal{C}}{\varepsilon^{7/3}} , \quad (13)$$

where \mathcal{C} is independent of ε and it depends on parameters of underlying theory. The expression for the transition rate thus can be written in the form

$$\frac{d\Gamma}{dA} = \frac{\mathcal{C}}{\varepsilon^{7/3}} \exp \left(-\frac{16\pi\mu^3}{3\varepsilon^2} \right) \frac{\tilde{\mathcal{Z}}_{12}}{\mathcal{Z}_1} , \quad (14)$$

with the path integral $\tilde{\mathcal{Z}}_{12}$ running only over the transverse variables ζ and z

$$\tilde{\mathcal{Z}}_{12} = \int \mathcal{D}\zeta \mathcal{D}z_1 \mathcal{D}z_2 \exp(-\tilde{S}_{12}) \quad (15)$$

and involving only the quadratic in these variables part of the action (11)

$$\tilde{S}_{12} = \frac{4\pi}{3}\varepsilon L^3 + \frac{16\pi\mu^3}{3\varepsilon^2} + \frac{\mu}{2} \int d\Omega g^{ab} \partial_a \zeta \partial_b \zeta + \frac{\varepsilon}{2} \int_R^L r^2 dr d\Omega g^{ij} \partial_i z \partial_j z. \quad (16)$$

In the same quadratic approximation the partition function \mathcal{Z}_1 for the trivial configuration is given by

$$\mathcal{Z}_1 = \int \mathcal{D}z \exp(-S_1) \quad (17)$$

with S_1 given by Eq.(12) and the integral running over all the functions vanishing at the space-time boundary: $z(L, \theta, \varphi) = 0$.

It was shown in [7] that the calculation of the ratio of this type of the partition functions can be reduced to a calculation of partition functions associated with boundary only. It is clear what is meant by the boundary partition function for bounce configuration. It is possible to define a similar object for the flat wall configuration in the following way. Although there is no relation between the flat wall configuration and the sphere with radius $R = \frac{2\mu}{\varepsilon}$, one can calculate the partition function \mathcal{Z}_1 by first fixing the transverse variable z at $r = R$: $z(R, \theta, \varphi) = \zeta(\theta, \varphi)$ and separating the integration over the bulk variables, hence introducing by hands the boundary for trivial configuration. As a result one gets the ratio in the form

$$\frac{\mathcal{Z}_{12}}{\mathcal{Z}_1} = \frac{\mathcal{Z}_{12(\text{boundary})}}{\mathcal{Z}_{1(\text{boundary})}}, \quad (18)$$

with boundary partition functions given by

$$\mathcal{Z}_{12(\text{boundary})} = \int \mathcal{D}\zeta \exp \left[-\frac{\mu}{2} \int d\Omega \zeta \Delta^{(2)} \zeta + \frac{\varepsilon R^2}{2} \int d\Omega \zeta \partial_r z_c \Big|_{r=R} \right], \quad (19)$$

and

$$\mathcal{Z}_{1(\text{boundary})} = \int \mathcal{D}\zeta \exp \left[\frac{\varepsilon R^2}{2} \int d\Omega \zeta \partial_r z_c \Big|_{r=R} \right], \quad (20)$$

with the function $z_c(r, \theta, \varphi)$ satisfying the Laplace equation $\Delta z_c = 0$ with the boundary conditions

$$z_c(R, \theta, \varphi) = \zeta(\theta, \varphi), \quad z_c(r = L) = 0. \quad (21)$$

The operator $\Delta^{(2)}$ is the angular part of the Laplace operator in 3d (the Laplace operator on a sphere).

One can find the complete set of these functions by expanding the boundary function $\zeta(\theta, \varphi)$ in the series of spherical harmonics

$$\zeta(\theta, \varphi) = \sum_{l,m} A_{lm} Y_{lm}(\theta, \varphi) . \quad (22)$$

In each of the partial waves these functions are then found as

$$z_c(r, \theta, \varphi) = A_{lm} \frac{R^{l+1}}{r^{l+1}} Y_{lm}(\theta, \varphi) . \quad (23)$$

Substituting these functions to the equations (19) and (20) and performing integration over the amplitudes A_{lm} yields

$$\mathcal{Z}_{12(\text{boundary})} = \mathcal{N} \prod_{l=0}^{\infty} \left(\frac{1}{\mu l(l+1) + \varepsilon R(l+1)} \right)^{(2l+1)/2} \quad (24)$$

and

$$\mathcal{Z}_{1(\text{boundary})} = \mathcal{N} \prod_{l=0}^{\infty} \left(\frac{1}{\varepsilon R(l+1)} \right)^{(2l+1)/2} . \quad (25)$$

4 Regularization

Clearly, each of the formal expressions (19) and (20) contains a divergent product, and their ratio is also ill defined, so that our calculation requires a regularization procedure that would cut off the contribution of harmonics with large l . In order to perform such regularization we use the standard Pauli-Villars method and introduce regulator fields Z_α with the weight factors C_α such that

$$\sum_{\alpha} C_{\alpha} = 1, \quad \sum_{\alpha} C_{\alpha} M_{\alpha}^n = 0, \quad (26)$$

for any n less than some finite number. The action corresponding to the quadratic part of the Nambu-Goto expression (5) for small Z_α :

$$\tilde{S}_{12} = \frac{\mu}{2} \int d\Omega g^{ab} \partial_a \zeta_\alpha \partial_b \zeta_\alpha + \frac{\varepsilon}{2} \int_R^L r^2 dr d\Omega \left(g^{ij} \partial_i Z_\alpha \partial_j Z_\alpha + M_\alpha^2 Z_\alpha^2 \right) . \quad (27)$$

with M_α being each regulator mass, which physically should be understood as satisfying the condition $M_\alpha \ll M_0$ and still being much larger than the relevant scale in the discussed problem, in particular $M_\alpha R \gg 1$. The regularized expression for the ratio of the boundary terms in \mathcal{Z}_{12} and \mathcal{Z}_1 thus takes the form

$$\frac{\mathcal{Z}_{12(\text{boundary})}}{\mathcal{Z}_{1(\text{boundary})}} \longrightarrow \mathcal{R} = \left[\frac{\mathcal{Z}_{12(\text{boundary})}}{\mathcal{Z}_{12(\text{boundary})}^{(R)}} \right] \left[\frac{\mathcal{Z}_{1(\text{boundary})}}{\mathcal{Z}_{1(\text{boundary})}^{(R)}} \right]^{-1} , \quad (28)$$

where we introduced the notation \mathcal{R} for the regularized ratio, and the regulator partition functions $\mathcal{Z}_{12(\text{boundary})}^{(R)}$ and $\mathcal{Z}_{1(\text{boundary})}^{(R)}$ are determined by the same expressions as in Eqs.(19) and (20) with function z_c being replaced regulator functions counterparts $Z_{\alpha c}$ which still satisfy the boundary conditions similar to (21):

$$Z_{\alpha c}(R, \theta, \varphi) = \zeta_{\alpha}(\theta, \varphi) , \quad (29)$$

but are the solutions of the Helmholtz rather than the Laplace equation $(\Delta - M_{\alpha}^2)Z_{\alpha} = 0$. The solutions of the Helmholtz equation fall off exponentially at the scale determined by M_{α} , and for our purposes only the behavior near the sphere $r = R$ is needed. For this reason we write the equation for the radial part of the l -th angular harmonic as

$$Z_l'' + \frac{2}{r} Z_l' - \frac{l(l+1)}{r^2} Z_l - M^2 Z_l = 0 , \quad (30)$$

where for a time being we have omitted the indices α and c for regulators. Introducing new function \tilde{Z}_l : $Z_l(r) = \sqrt{R/r} \tilde{Z}_l(r)$ we can rewrite previous equation in the form

$$\tilde{Z}_l'' + \frac{1}{r} \tilde{Z}_l' - \frac{(l+1/2)^2}{r^2} \tilde{Z}_l - M^2 \tilde{Z}_l = 0 . \quad (31)$$

One can now write the radial coordinate as $r = R + x$, and treat the parameter (x/R) as small, since the scale for the variation of the solution is $x \sim 1/\sqrt{M^2 + (l+1/2)^2/R^2}$. This approach yields an expansion of the regulator action associated with the boundary at $r = R$ in powers of $1/\sqrt{(MR)^2 + (l+1/2)^2}$. With the accuracy required in the present calculation, the (normalized to one at $r = R$) solution to Eq.(30) is found in the first order of expansion in (x/R) as

$$\tilde{Z}_l(R+x) = \sqrt{\frac{R}{r}} \left(1 - \frac{1}{2} \frac{(MR)^2}{(MR)^2 + (l+1/2)^2} \frac{x}{R} \right) \exp \left(-\sqrt{(MR)^2 + (l+1/2)^2} \frac{|x|}{R} \right) . \quad (32)$$

Using the form of the solutions for the harmonics of the regulator field given by Eq.(32) and the expressions (19) and (20), one can write the regularized ratio of the boundary partition functions (28) as

$$\mathcal{R} = \prod_{l=0}^{\infty} \left[\frac{l(l+1) + 2\sqrt{M_{\alpha}^2 R^2 + \left(l + \frac{1}{2}\right)^2} + 1}{(l+1)(l+2)} \right]^{(2l+1)C_{\alpha}/2} \times$$

$$\begin{aligned}
& \prod_{l=0}^{\infty} \left[\frac{l+1}{\sqrt{M_{\alpha}^2 R^2 + \left(l + \frac{1}{2}\right)^2} + \frac{1}{2}} \right]^{(2l+1) C_{\alpha}/2} \times \\
& \prod_{l=0}^{\infty} \left[1 + \frac{M_{\alpha}^2 R^2}{\left(M_{\alpha}^2 R^2 + \left(l + \frac{1}{2}\right)^2\right) \left(l(l+1) + 2\sqrt{M_{\alpha}^2 R^2 + \left(l + \frac{1}{2}\right)^2} + 1\right)} \right]^{(2l+1) C_{\alpha}/2}
\end{aligned} \tag{33}$$

where we took into account that $R = 2\mu/\varepsilon$.

5 Calculating the products

Each of the products in Eq.(33) is finite at a finite M and can be calculated separately. Instead of calculating the product directly, we can use the relation

$$\prod_l F_l = \exp \left(\sum_l \ln F_l \right), \tag{34}$$

and calculate the sum. We start from the third product. The expression under the sign of product is of the form $1 + g(l)$, with $g(l)$ close to 0 for any l , since it behaves as M_{α}^{-1} . Hence we can leave only the first term in the expansion of the logarithm $\ln(1+x) = x + O(x^2)$. Thus, we need to find the sum

$$S_3 = \frac{1}{2} \sum_{l=0}^{\infty} \sum_{\alpha} C_{\alpha} (2l+1) M_{\alpha}^2 R^2 \left[M_{\alpha}^2 R^2 + \left(l + \frac{1}{2}\right)^2 \right]^{-1} \left[l(l+1) + 2\sqrt{M_{\alpha}^2 R^2 + \left(l + \frac{1}{2}\right)^2} + 1 \right]^{-1}. \tag{35}$$

The sums associated with the three products are readily calculable with the help of the Euler-Maclaurin summation formula and the result for the regularized ratio has the form

$$\mathcal{R} = \exp \left[\frac{1}{2} M^2 R^2 \ln MR + MR \ln MR + \ln MR \right] \tag{36}$$

where $M^n \ln M = \sum_{\alpha} C_{\alpha} M_{\alpha}^n \ln M_{\alpha}$, for any n . The expression for \mathcal{R} contains an essential dependence on the regulator mass parameter M . We will show, however, that all such dependence in the phase transition rate can be absorbed in renormalization of the parameter μ in the leading semiclassical term. Although, it may appear that there is a problem with the term $MR \ln M$, which is not proportional to R^2 (the area of the interface) and thus is not the coefficient in front of tension μ . But this behavior is only due to the particular dependence of the radius R on μ and ε .

6 Renormalization of μ

The parameter μ is defined in the action (5) as the coefficient in front of the area of the interface between the world space of the wall and empty space. Generally this parameter gets renormalized by the quantum corrections, and in order to find such renormalization at the level of first quantum corrections, one needs to perform the path integration using the quadratic part of the action around a configuration, in which the area of the boundary is an arbitrary parameter. For a practical calculation of this effect we consider a Euclidean space configuration, with the wall lying flat in (x, y) plane, and the interface being at $y = 0$. Thus, the area of the world surface of the boundary is XT , with T and X being the size of the world volume of the wall in the (t, x) plane. The Gaussian path integral over the transverse coordinates $z(t, x, y)$ is then to be calculated. We use the notation $\zeta(t, x)$ for the transverse shift of the boundary and expand it in the Fourier series

$$\zeta(t, x) = \sum_{\vec{k}} a(\vec{k}) e^{i\vec{k}\vec{x}}. \quad (37)$$

A similar expansion applies to the regulator boundary function $\zeta_R(t, x)$. The part of the effective action associated with the boundary is determined by the functions $z_c(t, x, y)$ for the transverse shift of the string and the corresponding regulator functions $Z_c(t, x, y)$ that satisfy the equations

$$\Delta z_c = 0 \quad \text{and} \quad (\Delta - M_\alpha^2) Z_c = 0, \quad (38)$$

and the boundary conditions

$$z_c(t, x, 0) = \zeta(t, x), \quad Z_c(t, x, 0) = \zeta_R(t, x) \quad (39)$$

as well as

$$z_c(t, x, \pm\infty) = Z_c(t, x, \pm\infty) = 0. \quad (40)$$

One can readily find these functions for each harmonic of the boundary values ζ and ζ_R

$$z_c^{\vec{k}}(x, t, y) = e^{-\sqrt{k^2} |y|} e^{i\vec{k}\vec{x}}, \quad (41)$$

and

$$Z_c^{\vec{k}}(x, t) = \exp \left[-|y| \sqrt{k^2 + M_\alpha^2} \right] e^{i\vec{k}\vec{x}}. \quad (42)$$

In order to separate the boundary effect in the path integral around the considered configuration from the bulk effects, we again divide it by the path integral around the configuration

where the whole world volume is occupied by the wall. Such division results, as previously, in the cancellation of the bulk contributions, and the remaining part of the effective action associated with the boundary is written in terms of the regularized path integral over the boundary function ζ as

$$\begin{aligned} \mu_R \mathcal{A} = \mu \mathcal{A} - \\ \ln \frac{\int \mathcal{D}\zeta \exp \left\{ -\frac{1}{2} \int d\Omega \left[\mu g^{ab} \partial_a \zeta \partial_b \zeta - \varepsilon \zeta \partial_y z_c(x, t, y)|_{y \rightarrow +0} \right] \right\}}{\int \mathcal{D}\zeta_R \exp \left\{ -\frac{1}{2} \int d\Omega \left[\mu g^{ab} \partial_a \zeta_R \partial_b \zeta_R - \varepsilon \zeta \partial_y Z_c(x, t, y)|_{y \rightarrow +0} \right] \right\}} - \\ \ln \frac{\int \mathcal{D}\zeta \exp \left\{ -\frac{1}{2} \varepsilon \zeta \partial_y z_c(x, t, y)|_{y \rightarrow +0} \right\}}{\int \mathcal{D}\zeta_R \exp \left\{ -\frac{1}{2} \varepsilon \zeta_R \partial_y Z_c(x, t, y)|_{y \rightarrow +0} \right\}}, \end{aligned} \quad (43)$$

where $\mu_R = \mu + \delta\mu$ is the renormalized mass parameter. The correction to μ can thus be written in the form

$$\begin{aligned} \delta\mu &= \frac{1}{2} \int \frac{d^2 k}{(2\pi)^2} \left[\ln \left(k^2 + \frac{\varepsilon}{\mu} \sqrt{M^2 + k^2} \right) - \ln \left(k^2 + \frac{\varepsilon}{\mu} \sqrt{k^2} \right) \right] - \\ &\frac{1}{4} \int \frac{d^2 k}{(2\pi)^2} \left[\ln (M^2 + k^2) - \ln k^2 \right] = \\ &\frac{M^2}{8\pi} \ln MR + \frac{M\varepsilon}{8\pi\mu} \ln MR + \frac{\varepsilon^2}{16\pi\mu^2} \ln MR. \end{aligned} \quad (44)$$

It should be mentioned here, that result for the renormalized parameter μ_R would not change if we considered the wall not with a topology of a plane, but rather of a cylinder, namely, considering periodic boundary conditions in one direction.

7 Results and conclusion

Collecting all terms together we find the rate of the process. It is clear that the result for each $d - 3$ transverse dimensions factorizes, thus we have the expression for the rate in the form

$$\frac{d\Gamma}{dA} = \frac{\mathcal{C}}{\varepsilon^{7/3}} \mathcal{R}^{d-3} \exp \left(-\frac{16\pi\mu^3}{3\varepsilon^2} \right), \quad (45)$$

where μ is bare (non-renormalized) tension, and regularized ratio \mathcal{R} given by (36). Taking into account that each of the transverse dimensions contributes additively to $\delta\mu$ and the interface is a sphere with area

$$\mathcal{A} = 4\pi R^2, \quad (46)$$

and expressing the bare μ through the renormalized one: $\mu = \mu_R - \delta\mu$, one readily finds that the dependence on the regulator mass M cancels in the transition rate, and one arrives at the formula given by Eq.(4).

Thus we obtained the result similar to the decay of a string, when the effect of all extra transverse dimensions results only in the renormalization of parameter μ associated with the interface. It should be mentioned, however, that the result for a string was, actually, obtained for a transition between two states of a string with different tensions. Here we considered decay of a wall (transition into nothing). For the calculation used it is impossible to preserve finite terms, but only proportional to some power of the regulator mass parameter M .

Having calculated the probability rate for a decay of one- and two- dimensional objects, e.g. string and domain wall, it is tempting to assume that the situation is somewhat similar for the decay of an object of arbitrary dimensionality. But it is not true already for the decay of tree- and four- dimensional objects, where the dependence of the result on regulator mass is substantial even after renormalization of a parameter associated with an interface. That dependence demands introduction of new terms into initial action, which corresponds to nonrenormalizability of the effective ‘low-energy’ theory.

Acknowledgments

This work is supported in part by the DOE grant DE-FG02-94ER40823.

References

- [1] M. A. Shifman, Prog. Part. Nucl. Phys. **39**, 1 (1997).
- [2] E. Witten, Phys. Rev. Lett. **81**, 2862, 1998.
- [3] E. Witten, JHEP **9807**, 006 (1998).
- [4] M. B. Voloshin, I. Y. Kobzarev and L. B. Okun, Sov. J. Nucl. Phys. **20**, 644 (1975) [Yad. Fiz. **20**, 1229 (1974)].
- [5] S. R. Coleman, Phys. Rev. D **15**, 2929 (1977) [Erratum-ibid. D **16**, 1248 (1977)].
- [6] M. B. Voloshin, Phys. Lett. **B599**, 129 (2004).

- [7] A. Monin and M. B. Voloshin, Phys. Rev. D **78**, 065048 (2008).
- [8] J. Schwinger, Phys. Rev. **82**, 664 (1951).
- [9] M. M. Forbes and A. R. Zhitnitsky, JHEP **0110**, 013 (2001).